Closing Wed night: HW_1A, 1B, 1C

Entry Tasks:

(a) $f''(x) = 15\sqrt{x}$,
$f(1) = 0, f(4) = 1$
Find $f(x)$.

(b) Ron steps off the 10 meter high dive at his local pool. Find a formula for his height above the water. (Assume his acceleration is a constant 9.8 m/s^2 downward)

5.1 Defining Area

Motivation:

Calculus is based on limiting processes that “approach” the exact answer to some instantaneous rate question.

In Calculus I, you defined

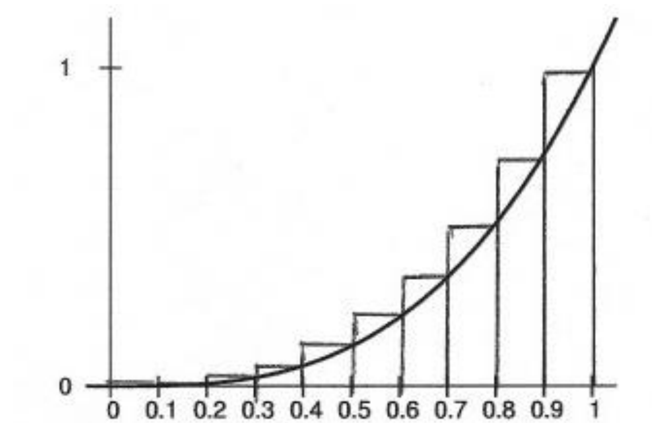
$f'(x)$ = `slope of the tangent at x’

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

In Calculus II, we will see that antiderivatives are related to the area `under’ a graph

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

Visual:



Riemann sums set up:

We are going to build a procedure to get better and better approximations of the area “under” $f(x)$.

1. Break into *n* equal subintervals.

   $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

2. Draw *n* rectangles; use function.

   Area of each rectangle =

   (height)(width) $= f(x_i^*)\Delta x$

3. Add up rectangle areas.

Example: Approximate the area under $f(x) = x^3$ from x = 0 to x = 1 using n = 3 subdivisions and *right-endpoints* to find the height.

I did this again with 100 subdivisions, then 1000, then 10000. Here is the summary of my findings:

n	R_n	L_n
4	0.390625	0.140625
5	0.36	0.16
10	0.3025	0.2025
100	0.255025	0.245025
1000	0.25050025	0.24950025
10000	0.2499500025	0.2500500025

Right-Endpoint Pattern:

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}, \qquad x_i = 0 + i\,\frac{1}{n} = \frac{i}{n}$$

$$\text{Height of rect.} = f(x_i) = x_i^3 = \left(\frac{i}{n}\right)^3$$

$$\text{Area} = f(x_i)\Delta x = x_i^3 \Delta x = \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

$$\text{Sum} = \sum_{i=1}^{n} x_i^3 \Delta x = \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

$$\text{Exact Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

Definition of the Definite Integral

We define the exact area "under" f(x) from x = a to x = b curve to be

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x,$$

where $\Delta x = \frac{b-a}{n}$ and

$$x_i = a + i\Delta x.$$

We call this the definite integral of f(x) from x = a to x = b, and we write

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$